2011 FGCU Invitational Mathematics Competition

Thursday, December 8th, 2011

Precalculus Team Test

Directions

• You will have 4 minutes to answer each question.
• The scoring will be 16 points for a correct response in the 1st minute, 12 points for a correct response in the 2nd minute, 8 points for a correct response in the 3rd minute, 4 points for a correct response in the 4th minute. A sliding scale will be used.
• Once your hand has been raised with the answer sheet, it must stay up. If you put your hand down, your answer will be disqualified for that question.
• Your answer must be submitted on the official answer sheet that has been correctly filled out. Otherwise your answer will be disqualified.
• Your answer must be in the specific form that the question asks for.
Directions……

• If not otherwise noted, the answers should be in one of the following generally accepted forms:
  – Denominators rationalized
  – Simplest radical form
  – Fractions, improper fractions, or mixed numbers in simplest form
  – Equations should have integral coefficients in standard form

• No units are necessary
• Calculators are not allowed in any division except Statistics.
• Headphones, beepers, cell phones, or electronic devices are not permitted.
• Sunglasses and hats are not to be worn during the competition.

Are you ready?
1) Find \[ \begin{vmatrix} a & b \\ c & d \end{vmatrix} \] where

- \( a \) is the exact value of \( \tan \frac{\pi}{12} \)
- \( b \) is the solution to \( \left( \frac{\pi}{4} \right)^{2x+5} = \left( \frac{4}{\pi} \right)^{x+7} \)
- \( c \) is the exact value of \( 8 \cdot 2^{\log_{16} 81} \)
- \( d \) is the integer solution to \( \frac{\sqrt{x^2+9(x-3)}}{x-5} \leq 0 \)

2) Solve the equation.

\[ \log_2 x^2 = (\log_2 x)^2 \]
3) Find the exact value.
\[ \sin 10^\circ - \sin 80^\circ \tan 50^\circ \]

4) Find a rectangular equation for the plane curve defined by the parametric equations.
\[ x = 5 \tan \theta, \quad y = 4 \cot \theta \]
5) Solve the system for $x$ and $y$. Assume $a$ and $b$ are nonzero constants.

$$ax + by = \frac{a}{b}$$

$$x + y = \frac{1}{a}$$

6) Solve.

$$3^{5x-x^2-6} + 3^{5x-x^2-3} \geq 28$$
7) Let \( f(x) = \sin x \), \( g(x) = \cos x \), and \( h(x) = \tan x \).

Given the point \((x, \sqrt{3})\) on the circle \(x^2 + y^2 = 7\),
also lies on the terminal side of an angle \( \alpha \) in
quadrant II and the point \((-\frac{1}{3}, y)\) on the circle
\(x^2 + y^2 = 1\), also lies on the terminal side of an
angle \( \beta \) in quadrant III, find:

\[
f \left( \frac{\alpha}{2} \right) g \left( \frac{\alpha}{2} \right) - h^2 \left( \frac{\beta}{2} \right)
\]

8) Let \( f(x) = \cos x \), \( g(x) = \sin x \) and \( h(x) = \tan x \).

Find all \( x \) on \((-\frac{\pi}{2}, \frac{\pi}{2})\) such that \( f \circ g \circ h(x) = 1 \).
9) Find the exact value of the expression:

\[ \sin^2 \left( \frac{1}{2} \cos^{-1} \left( \frac{4}{5} \right) \right) \]

10) Given \( \log_b M = \cos 2\theta \) and \( \log_b N = \sin 2\theta \), evaluate \( \log_b MN^2 \).
11) Find all θ on \((0, \pi/2]\) such that the matrix is not invertible.

\[
\begin{pmatrix}
\sin\theta & \pi \sin\theta \\
\sin\theta & 0 \\
\sin\theta \pi & \csc\theta \\
\sin\theta & \pi \csc\theta
\end{pmatrix}
\]

12) \(P_1 = (-3, 4), P_2 = (0, -1)\) are two of the vertices of a right isosceles triangle. Given that \(a = d(P_1, P_2)\) is the length of one of the legs, \(b\) is the length of the other leg and \(c\) is the hypotenuse, find:

\[
\begin{pmatrix}
0 & a \\
b & c
\end{pmatrix}
\]