Directions

- You will have 4 minutes to answer each question.
- The scoring will be 16 points for a correct response in the 1st minute, 12 points for a correct response in the 2nd minute, 8 points for a correct response in the 3rd minute, 4 points for a correct response in the 4th minute. A sliding scale will be used.
- Once your hand has been raised with the answer sheet, it must stay up. If you put your hand down, your answer will be disqualified for that question.
- Your answer must be submitted on the official answer sheet that has been correctly filled out. Otherwise your answer will be disqualified.
- Your answer must be in the specific form that the question asks for.
Directions……
• If not otherwise noted, the answers should be in one of the following generally accepted forms:
  – Denominators rationalized
  – Simplest radical form
  – Fractions, improper fractions, or mixed numbers in simplest form
  – Equations should have integral coefficients in standard form
• No units are necessary
• Calculators are not allowed in any division except Statistics.
• Headphones, beepers, cell phones, or electronic devices are not permitted.
• Sunglasses and hats are not to be worn during the competition.

Are you ready?
1) Find the midpoint and length of the line segment whose endpoints are given by the solutions of the following systems of equations:

\[
\begin{align*}
3x - 4y &= 11 \\
2x + 3y &= -4
\end{align*}
\quad \quad \begin{align*}
3x - 2y &= -5 \\
4x + y &= 8
\end{align*}
\]
2) Use substitution to solve the equation:

\[ 3^{2x} + 2 \cdot 3^{x+3} + 3^7 = 3^{x+5} - 3^{x+4} \]
3) Find the length of the red line segment:

\[ f(x) = x^3 - x^2 - 4x + 3 \]

\[ g(x) = 2x + 3 \]
4) Find the LCM of the following numbers:

600 and 1260

Are you ready?
5) Let \( f(x) = 2x^3 - 3x^2 + 4x + 18 \). Find the equation of the line with slope \( \frac{1}{3} [f(2) - f(1)] \) that intersects \( f(x) \) at \( x = 3 \). Give your answer in slope-intercept form.
6) Give the quadratic function with zeros
   \[ x = 2 \pm \sqrt{5} \] and leading coefficient 3.

Are you ready?
7) Find the area of the shaded region in terms of $x$. Assume that the quadrilateral is square and the octagon is regular.

Are you ready?
8) If the volume of the *un-shaded* region of the cube is 48, find x. Assume that both shaded regions are cubes and that they share a corner.

\[ \text{Are you ready?} \]
9) Find five consecutive odd integers such that the first plus nine times the second minus seven times the third plus three times the fourth plus the fifth is five less than twice seventy.
10) Let
\[ a \star b = (a - b)^2 + 2ab \]
\[ a \bigcirc b = \frac{a^2 - b^2}{a^2 + b^2}. \]
Evaluate
\[
\frac{6 \bigcirc 4 \cdot \frac{4\sqrt{5}}{3} \ast [\sqrt{2} \bigcirc (-1)]}{\sqrt{7} \bigcirc \sqrt{6} \ast 3 \ast 3}
\]
11) You need to rent a car to drive from St. Louis, MO to San Diego, CA; an 1,800 mile trip. Car rental company A charges $45 per day for unlimited mileage. Car rental company B charges $24 per day plus $0.20 per mile. Car rental company C charges $17 per day for the first three days, then $85 dollars per day for each additional day. Driving for eight hours a day at an average speed of 45 miles per hour, which car rental company is the least expensive? Give the company and the cost of the least expensive car rental.

Are you ready?
12) The sum of the perimeter of the triangle and five times the perimeter of the rectangle is nine more than twice eighty. The sum of the perimeter of the rectangle and eight times the perimeter of the triangle is four more than the square of ten. Find $x$ and $y$. 

\[ x + 2y \]

\[ x + 2y \]

\[ x - 2y \]
13) Let $f(x) = x^2 - 4x + 4$, $g(x) = 3x - 5$.
Evaluate:

$$\sqrt{f(-1) - f(0)} - [g(2)]^2 + f(-2) \div f(-1) \cdot g(2)$$

**Are you ready?**
14) Determine the value of A so that the line whose equation is $3y + 12x = 3(3Ax + 2)$ is perpendicular to the line passing through the points $(1,-3)$ and $(-2,4)$.
15) If a coin is tossed \( k \) times, we expect approximately half of the outcomes to be heads. It can be demonstrated that a coin is unfair if after \( k \) tosses, the number of outcomes that result in heads, \( h \), satisfies

\[
\left| h - 0.5 \cdot k \right| \geq \frac{10}{6}.
\]

Describe the number of outcomes that determine an unfair coin that is tossed 100 times. Give integers for values of \( h \), and write your answer in interval notation.

**Answers**

1) (1,1), 6  
2) [3,4]  
3) \( 3\sqrt{5} \)  
4)12600  
5)\( y = 3x + 48 \)  
6) \( 3x^2 - 12x - 3 \)  
7)\( A = x^2 \left( 1 - \frac{\sqrt{3}}{2} \right) \)  
8) 2  
9) 17,19,21,23,25  
10) \( \frac{101}{4} \)  
11) Company C @ $221  
12) \( x = 3, \ y = 5 \)  
13) \( \sqrt{5} + \frac{7}{9} \)  
14) \( A = \frac{31}{21} \)  
15) \([0,41] \cup [59,100] \) or \([0,42] \cup (58,100] \)