Directions

• You will have 4 minutes to answer each question.

• The scoring will be 16 points for a correct response in the 1st minute, 12 points for a correct response in the 2nd minute, 8 points for a correct response in the 3rd minute, 4 points for a correct response in the 4th minute. A sliding scale will be used.

• Once your hand has been raised with the answer sheet, it must stay up. If you put your hand down, your answer will be disqualified for that question.

• Your answer must be submitted on the official answer sheet that has been correctly filled out. Otherwise your answer will be disqualified.

• Your answer must be in the specific form that the question asks for.
Directions……

- If not otherwise noted, the answers should be in one of the following generally accepted forms:
  - Denominators rationalized
  - Simplest radical form
  - Fractions, improper fractions, or mixed numbers in simplest form
  - Equations should have integral coefficients in standard form
- No units are necessary
- Calculators are not allowed in any division except Statistics.
- Headphones, beepers, cell phones, or electronic devices are not permitted.
- Sunglasses and hats are not to be worn during the competition.

1) Find the coordinates of the point one third of the way from P(-171, 26) to Q(3, -54).
2) Which of these can NOT be constructed with the normal restrictions of classical Greek methods – compass and straightedge?

A. Square  
B. Regular Pentagon  
C. Regular dodecagon  
D. Regular 17-gon  
E. All of the above may be constructed

3) An isosceles trapezoid of area 27 has one base length 12 and altitude 2. Find the length of one of the slanted sides.
4) Equilateral triangles are constructed as shown on each side of a regular octagon of side 2. Find the area of the star shape.

[Diagram of a regular octagon with equilateral triangles constructed on each side]

5) A frustum of a square-based pyramid is being filled at the rate of 26 cubic feet per 10 minutes. The top and bottom of the frustum are squares of sides 6 feet and 15 feet. Its altitude is 12 feet. How long will it take to fill the volume of the frustum? (answer in hours)

[Hint: pyramid volume = \((1/3)hB\)]
6) Isosceles triangle AEF with area $\frac{21}{2}$ is constructed as shown in square ABCD with side 5. Find the value(s) of $x$.

![Diagram of isosceles triangle AEF within square ABCD]

7) What is the minimum number of sides a convex $n$-gon must have so that the number of diagonals it has is larger than the total number of degrees of its interior angles?
8) Name the missing quadrilateral:

- quadrilateral
- parallelogram
- square
- rhombus
- trapezoid
- rectangle
- isosceles trapezoid

9) ABCDEF is a regular hexagon of side 2.

Centered on each side is constructed an equilateral triangle of side 1.

Find the area of the star shape.
10) Characterize figure ABCD (refer to next slide):

A. (just a) quadrilateral
B. trapezoid
C. parallelogram
11) A tree at 4 pm casts a 27.3 ft shadow. At the same moment a yardstick casts a shadow that is 8.4 inches. How tall is the tree? (answer in feet)

12) I stuck a foot-long ruler two inches into the ground, intending to measure the height of my tree, then went inside for a drink. When I came outside the ruler’s shadow was 8 inches. But the tree’s shadow fell on a hedge, so I couldn’t measure it precisely. The hedge is 36 ft from the tree, 2 ft wide, and 5 ft high. Can you find reasonable limits for the height of my tree? (answer in feet)
13) In “taxi-cab geometry” all lines must be either horizontal or vertical (no diagonals permitted), and all line segments are integral in length (no fractional lengths permitted). In this geometry, if a triangle – not a rectangle – has a base of 10 and an altitude of 7, what are the minimum and maximum area the triangle may have?

14) Starting with $x_0 = 3$, how many stages are necessary so that $x_n$ exceeds 120. In other words, what is the smallest $n$ can be so that $x_n$ is greater than 120. (Note: figure not drawn to scale. Each triangle is a right triangle. $x_0$ is the leg of the first triangle and the $x_1$ is the leg of the second triangle and so on.)