Directions

- You will have 4 minutes to answer each question.
- The scoring will be 16 points for a correct response in the 1st minute, 12 points for a correct response in the 2nd minute, 8 points for a correct response in the 3rd minute, 4 points for a correct response in the 4th minute. A sliding scale will be used.
- Once your hand has been raised with the answer sheet, it must stay up. If you put your hand down, your answer will be disqualified for that question.
- Your answer must be submitted on the official answer sheet that has been correctly filled out. Otherwise your answer will be disqualified.
- Your answer must be in the specific form that the question asks for.
Directions……

• If not otherwise noted, the answers should be in one of the following generally accepted forms:
  – Denominators rationalized
  – Simplest radical form
  – Fractions, improper fractions, or mixed numbers in simplest form
  – Equations should have integral coefficients in standard form
• No units are necessary
• Calculators are not allowed in any division except Statistics.
• Headphones, beepers, cell phones, or electronic devices are not permitted.
• Sunglasses and hats are not to be worn during the competition.

1) What is the value of

\[ 4 \ln(e^{10^6}) - (2014)(1986) \]
2) The sides of a triangle are in the ratio 4:5:6. Determine the secant of the largest angle.

3) The domain of the function

\[ f(x) = \sqrt{\frac{3\pi}{2} - \cos^{-1}(x + 2)} \] can be written as the closed interval [a, b] for real numbers a and b. Find the value of a + b.
4) What is the value of \( \left\lfloor \frac{\log_2 \pi^3}{\log_2 10} \right\rfloor \) where \( \lfloor x \rfloor \) represents the greatest integer less than or equal to \( x \)?

5) If \( [x] \) is the least integer greater than or equal to \( x \) evaluate:
\[
[ \log_3 (\sqrt{13} + \sqrt{14} + \sqrt{15}) + \log_3 (\sqrt{13} - \sqrt{14} + \sqrt{15}) + \ldots ]
\]
6) The equation $\sin(2\theta) = 2\cos^2\theta$ has solutions

\[
\frac{a\pi}{4}, \frac{b\pi}{4}, \frac{c\pi}{4}, \frac{d\pi}{4}
\]

on the interval $[0, 2\pi)$ with $a < b < c < d$.

Find $ad - bc$.

7) The graph of $r = 4\sin\theta$ meets the line

$y = x + 2$ in quadrant II at point A.

Point B lies on the circle at $(0, k)$ for $k > 0$.

Find the area of the sector bounded by $AO$, $BO$ and arc AB if O is the center of the circle.
8) Solve for $x$:

$$\log_3 \left( \frac{\sqrt[3]{729} \sqrt[3]{27 \sqrt{729}}}{\frac{1}{2} \frac{2}{5}} \right) = 2^{0^4}$$

9) Determine the period of the function

$$f(x) = \sin^4(2x)$$
10) A rectangle whose width is two times its length is inscribed inside the circle given by \( x^2 - 2x + y^2 - 6y = 90 \). Find the area of the rectangle.

11) Find the sum of the solutions on the interval \([0, \pi]\) of the equation \( \sin \theta = \cos(2\theta) \).
12) Two campers are on opposite sides of a tree. The tree makes a right angle with the ground and the campers are 160 feet apart along the ground. The campers' angles of elevation to see the top of the tree are 45 and 60 degrees. Determine the height (in feet) of the tree.

13) Evaluate:
\[ \ln\left(\cot^{-13} + \cot^{-17} + \cot^{-15} + \cot^{-18}\right) \].
14) Solve: \( \sqrt[3]{(x + 1)^2} = 2\sqrt[3]{x + 1} + 3 \)

15) Let \( f(x) \) and \( g(x) \) be functions defined by
\[
f(x) = \{(-5, -9), (3, -2), (8, 1)\} \quad \text{and} \quad g(x) = \{(-9, 3), (1, 4), (2, 1)\} .
\]
Find \( (g \circ f)(x) \).